Full-Wave Analysis and Model-Based Parameter Estimation Approaches for Y-matrix Computation of Microwave Distributed RF Circuits

V. Chtchekatourov, F. Coccetti, P. Russer
Institut für Hochfrequenztechnik, Technische Universität München
Arcisstrasse 21, D-80333 Munich, Germany, e-mail: chtcheka@hft.ei.tum.de

Abstract **Distributed microwave circuits are analyzed in time domain using the Transmission Line Matrix (TLM) method. System Identification (SI) and Spectral Analysis (SA) methods are used to compute S–, Y–parameters. Model–based approaches to calculate the Y–parameters from truncated signals are presented. The extracted models are compared with the running TLM analysis in real–time. By this way a stop criterion for the TLM computation is obtained. Compared with ordinary time–domain simulation the presented method of admittance parameter computation yields a reduction of computation time. The behavior of a microswitch realized via micromachining technique was investigated by using the TLM and the SA (SI) methods.**

I. Introduction

Fourier transformation (FT), high–resolution spectral analysis and system identification methods have been applied to extract the scattering parameters from the TLM [1, 2] time–domain system response [3]. Some model–based parameter estimation methods, especially the autoregressive–moving average (ARMA) method and the autoregressive (AR)–based method, were used to estimate **S**–parameters and to extrapolate the TLM computed time–domain signals [3]. These methods have been successfully applied to the analysis of microstrip antennas and microwave integrated circuits.

In our paper we present a method for direct computation of the multiport admittance parameters from the truncated time–domain impulse responses, which are computed via TLM. The parametrical and non–parametrical methods, introduced in this paper, are based on the second– and third–order statistics for the parameter estimation of exponentially damped harmonic signals in the presence of noise [4]. Linear reciprocal reactive multiports my be described by a canonical Foster representation (AFM) [5]. For the lossy case a semi–empirical extension of the Foster representation has been given in [6]. The 3D TLM simulation with subsequent equivalent circuit generation has been already applied to planar circuits, multichip modules, microwave microelectromechanical (MEMS) capacitive switches and microstrip filters [6, 7]. For structures exhibiting a lot of details and low losses the computation time may be extremely long. So the calculation of slowly decaying time–responses should be discontinued as soon as the model parameters can be determined with the desired accuracy. This will be the case if an accurate extrapolation of the time response of the signal is possible. The parameters of the equivalent AFM models and their updates can be estimated in real–time during the TLM simulation. The validation of the permanently updated models and the real–time error estimation allows to generate a "stop signal" for the computation process. This yields minimum computation time for a given error limit.

We have applied TLM simulation in combination with the described model–based parameter estimation approach to the simulation of microwave distributed RF circuits. The pure TLM analysis is time consuming because of the slowly decaying impulse responses. The extraction of the poles and residues has been performed in time–domain using SI and high–resolution SA methods. The use of the above–mentioned methods reduces the required time for the TLM simulation and therefore makes the numerical analysis much more efficient and suitable for optimization procedures. In additional high-order statistic (HOS) method for parameter estimation was applied to reduce all undesired Gaussian band–limited processes. A good coincidence of the data received from pure TLM simulation and data received from TLM simulation in connection with parameter estimation methods has been obtained for the investigated circuites.

II. Model–based Y–parameters Estimation

SI approaches, which analyze exciting impulses and corresponding transient responses simultaneously, or

SA methods for the separate analysis can be used to find $\mathbf{Y}$–parameters and to calculate the AFM [8, 9, 11]. Furthermore, according to the singularity expansion method by Baum [10], a driven part of a system response can be separated from a transient part. This allows us to calculate $\mathbf{Y}$–parameters and construct AFM by applying SA methods for truncated signals as it is shown in Fig.1.

From all SA methods Prony's approach [9] seems to be the most obvious way to find the parameters of AFM. The method seeks to fit a deterministic exponential model $\hat{s}$ to the TLM simulated data s_k:

$$\hat{s}_k = \sum_{n=1}^{d} \underbrace{|h_n| \exp(j\phi_n)}_{h_n} \underbrace{\exp[(\alpha_n + 2\pi f_n)k]}_{z_n} \quad , \tag{1}$$

where $k = 0, 1, \ldots,$ N–1 is the samples index, n is the order of the model, d is the number of poles, h_n is the complex amplitude of the pole z_n, and ϕ_n, α_n, f_n are the initial phases, damping factors and frequencies respectively. Parametrical methods and non–parametrical method to find z_n can be applied in the first step of Prony's method. The Least–squares parametrical estimation method is usually used for the determination of complex coefficients in ARMA or in AR models in order to get z_n. From K data records of N samples each, the second–order moment (in this case $v = 2$)

$$\hat{c}_v(\tau_1, \ldots, \tau_{v-1}) = \frac{1}{K} \sum_{i=1}^{K} \hat{c}_v^{(i)}(\tau_1, \ldots, \tau_{v-1}) =$$

$$= \frac{1}{K} \sum_{i=1}^{K} \frac{1}{N} \sum_{k=k1}^{k2} s^{(i)}(k) s^{(i)}(k+\tau_1) \cdots s^{(i)}(k+\tau_{v-1}) \tag{2}$$

$$k_1 = \max(0, -\tau_1, \ldots, -\tau_{n-1})$$
$$k_2 = \min(N-1, N-1-\tau_1, \ldots, N-1-\tau_{n-1})$$

is used to form a data covariance matrix $\mathbf{C}$. The vector of AR model coefficients $\mathbf{a} = (a_0 \ldots a_d)^T$ is calculated with prediction error ρ from the system of linear equations: $\mathbf{C} \cdot \mathbf{a} = \rho$. By means of polynomial factorization of $A(z) = \sum_n a_n z^{-n}$ the poles z_n are calculated. The matrix–shifting methods as ESPRIT, MUSIC or Pencil–of–Function (POF) are based on an analysis of covariance or data matrix subspaces [9] to get a set of z_n. So, for POF approach the column elements of the covariance matrix $\mathbf{C}$ build a set of information vectors $\mathbf{y}_0, \mathbf{y}_1, \cdots, \mathbf{y}_{M-1}$. Using this, the matrices $\mathbf{Y}_0$ and $\mathbf{Y}_1$ are defined as $\mathbf{Y}_0 = [\mathbf{y}_0, \mathbf{y}_1, \cdots, \mathbf{y}_{M-2}]$, and $\mathbf{Y}_1 = [\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_{M-1}]$. z_n, in this case, are the generalized eigenvalues of the matrix pencil problem $\mathbf{Y}_0 - z\mathbf{Y}_1$. The poles can be evaluated via $\mathbf{z} = \mathbf{D}^{-1}\mathbf{U}^{\mathrm{H}}\mathbf{Y}_1\mathbf{V}$, where $\mathbf{U}$, $\mathbf{D}$ and $\mathbf{V}$ are the components of the singular value decomposition of $\mathbf{Y}_0$. For noisy data one should choose

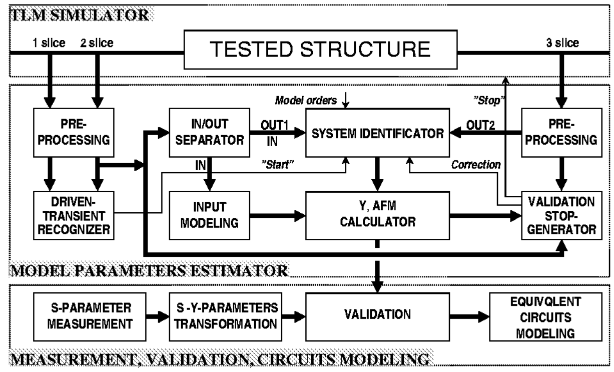

Figure 1: The general structure of applying SA and SI methods for $\mathbf{Y}$–parameters estimation and AFM construction

$\sigma_0, \cdots, \sigma_{S-1}$ to be the S largest singular values of $\mathbf{Y}_0$. $\mathbf{U}$, $\mathbf{D}$ and $\mathbf{V}$ correspond now to the signal subspace of the data. At the last step of Prony's method the estimation of correspondent residues h_n is made by solving the linear equations, built from Vandermonde poles matrix $\mathbf{Z}$: $\left(\bar{\mathbf{Z}}^T\mathbf{Z}\right)\mathbf{v} = \bar{\mathbf{Z}}^T\mathbf{s}$. The accuracy of the model (1) depends on d and should be optimized. If the bandwidth of the input signals is not sufficiently high to cover the natural spectrum of a system or if the investigated system has fast decaying components, the transient time–separation will be impossible and both input and output modeling or SI methods should be applied. The components of the multiport admittance matrix $\mathbf{Y}(z)$ for input voltages $U(z)$ and output currents $I(z)$ of a system can be found via

$$Y(z) = \frac{I(z)}{U(z)} = \underbrace{\frac{1 + \sum_{k=1}^{d} a_k z^{-k}}{1 + \sum_{l=1}^{q} b_l z^{-l}}}_{ARMA} \Longleftrightarrow \underbrace{\sum_{r=1}^{p} \frac{v_r}{1 - z_r z^{-1}}}_{Prony's Model} \quad . \tag{3}$$

SI approaches simultaneously determine the model parameters of microwave structures from multivariable input and output data. Due to the nature of considered devices, the problem of AFM construction is a common–mode system identification problem. From the huge set of SI models the output error (OE) model can be selected [11]. So the OE model for the case of $\mathbf{Y}$–parameters can be defined for input i and output j as:

$$I^j(z) = \frac{B^{i,j}(z)}{A(z)} U^i(z) + E(z) \quad . \tag{4}$$

The vector of unknown parameters (which builds $A(z)$ and $B^{i,j}(z)$) for two ports circuits ($i = 1, j = 1, 2$)

$$\mu = [1, a_1, \ldots a_d; b_1^{(1,1)}, \ldots b_{q1}^{(1,1)}; b_1^{(1,2)}, \ldots b_{q2}^{(1,2)}]^T \tag{5}$$

can be determined by linear or nonlinear Least–squares algorithms or, in the simplest way, by solving $F\mu = 0$, where F is a Gram matrix of the data.

There are some reasons to use higher–order statistics in electromagnetics: the cumulants of non–Gaussian processes carry higher–order statistical information about signals, third or bigger order cumulants of Gaussian processes are zero. Detection and identification of a non–minimum phase system, use of cumulants to suppress noise under certain conditions, detection and characterization of nonlinear properties in signals as well as identification of nonlinear system are the most interesting applications [4]. Let us consider the case of applying Prony's method with HOS [4]. The third or forth order cumulants are estimated from (2):

$$\hat{r}_2(\tau_1) = \hat{c}_2(\tau_1) \qquad \hat{r}_3(\tau_1, \tau_2) = \hat{c}_3(\tau_1, \tau_2) \tag{6}$$

$$\hat{r}_4(\tau_1, \tau_2, \tau_3) = \hat{c}_4(\tau_1, \tau_2, \tau_3) - \hat{c}_2(\tau_1)\hat{c}_2(\tau_3 - \tau_2) - $$
$$- \hat{c}_2(\tau_2)\hat{c}_2(\tau_3 - \tau_1) - \hat{c}_2(\tau_3)\hat{c}_2(\tau_2 - \tau_1). \tag{7}$$

The 1–D slice of the third–order or forth–order cumulant is used now to construct the data matrix for Prony's algorithm.

III. Application

The signal transmission and reflection behavior of the wide-band batch transfer microswitch [12] has been investigated using the TLM method. The series switch with the 60μm inner-conductor gap has been considered. The MEMS structure consists of a low-stress silicon nitride membrane carrying the actuation (4 in Fig.2) and the contact conducting pads (5 in Fig.2). The structure is mounted on top of a gold CPW transmission line (inner-conductor and slot widths are 30μm and 3.5μm respectively) realized on quartz substrate. The actuation pads (thickness 0.6μm) are made of polysilicon and connected to the biasing terminals by means of bonding bumps (3 in Fig.2). The gold contact pad (thickness 0.5μm) is located underneath the silicon nitride membrane (thickness 1μm) and is distant 5μm from the inner-conductor. The membrane realizes the electric isolation between the two conducting areas. In order to reduce the simulated TLM domain the bumps have been located at the external edge of the ground plane. The only off-state behaviour (membrane not actuated) has been simulated. The TLM model of the simulated structure (half of the real one with the magnetic wall 1 in Fig.2) exhibits $262 \times 71 \times 118$ nodes. A time discretization interval of $1.333^{-15}s$ was chosen for the TLM simulation. For direct TLM **Y**–parameter computation both parts of the circuit have been terminated by zero impedance (wall 2 in Fig.2). The input signal covers the frequency band of interest (up to $3 \cdot 10^{11}$Hz). The output time–domain currents were calculated at the various slices. For signal processing of the TLM simulation results, the sampling interval has been increased by up to two orders of magnitude. Parametrical Least–square (LS) and non–parametrical matrix–shifting Prony's (Pencil–of–Function, POF) methods were chosen for the modeling. Two different criteria – a threshold of the validation errors and unchanging of the found model parameters can be used to estimate the quality of the created models so to generate a "stop signal" for the TLM simulator. All found poles should be inside of the unit circle of the complex plane to avoid instability. The validation errors for different sample numbers, used for the AFM calculation, and for different number of AFM parameters, were investigated. The errors for all these methods don't change significantly, if the number of poles in the canonical Foster admittance model, exceeds some value. POF–Prony's approach gave us the smallest order threshold. Found with the low-order LS method poles position are more regularly distributed around the unit circuit in z–domain and for small model order they do not cover all significant signal components up to $3 \cdot 10^{11}$Hz (Fig.3). Low-order Matrix–shifting based methods put the poles in order and in this case yields to less calculation efforts to find the correct low frequency pole set. Also equivalent circuits modeling needs the pole positions of AFM placed in the relative low frequency range. With some assumption undesired Gaussian processes can be particularly suppressed by applying the higher–order statistics at the first step of Prony's method. The $\hat{r}_3(l, 0)$ 1–D slice was chosen and applied to create the signal data matrix. Applying of HOS in order to decrease the errors should be study together with measurement validation as it was shown in Fig.1. The calculation of the **Y**–parameters can be made from the measured **S**–parameters via

$$\mathbf{Y} = [\mathbf{q} + \mathbf{Sq}]^{-1}[\mathbf{q}^{-1} - \mathbf{Sq}^{-1}], \tag{8}$$

where **q** is the diagonal matrix formed by the square roots of the characteristic impedances of the ports. Using these methods we are expecting a further improvement of accuracy and also a reduction of the computational effort.

Note that low–order sequential SI algorithms can be used, whether the input signal bandwidth is not sufficiently high or the system components decay so fast that transient part separation is not possible. Otherwise, the matrix–shifting Prony's methods for truncated signals are applied for the AFM construction. The "stop signal" generation and the reduction of the total calculation TLM time for AFM construction strongly depends on the investigated structures.

E.g. by applying the POF–Prony's method for the used structures, the total time was reduced by about factor 5.

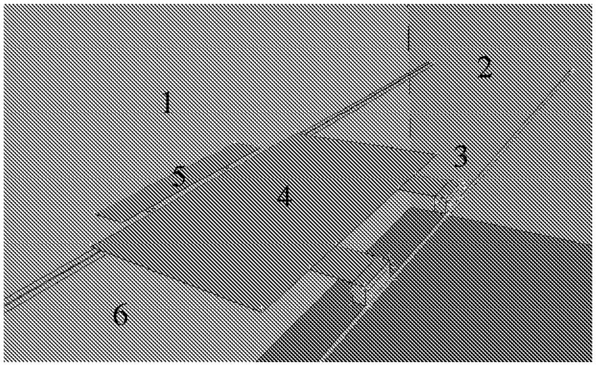

Figure 2: TLM space of the analyzed structure

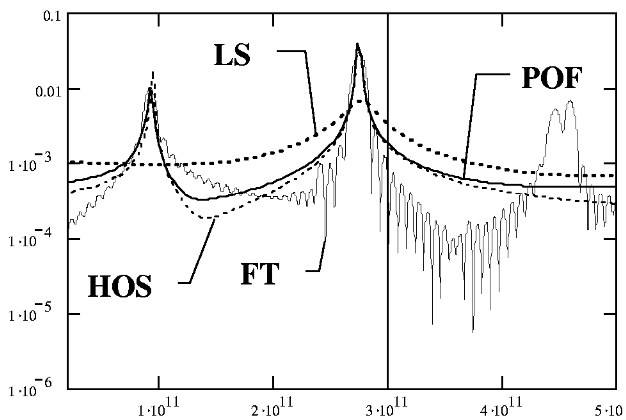

Figure 3: FT and low-order models of $|\mathbf{Y21(f)}|$ ($d_{POF} = d_{HOS} = d_{LS} = 6$)

IV. Conclusions and Outlook

We have presented SI and SA model–based methods for efficient computation of $\mathbf{S}-$ and $\mathbf{Y}-$parameters for distributed microwave circuits. The modeling is based on three–dimensional electromagnetic full–wave computation of the admittance parameters of distributed multiport circuits. Based upon these computations a lumped element model in a modified Foster representation is extracted using Prony's parametrical and non–parametrical methods. The parameter extraction is performed in real–time during the TLM simulation. The time–domain simulation is terminated if the model error passes a lower threshold. By this way minimum computation time for a specified error threshold can be achieved.
Applying the POF–Prony's method the total calculation time was reduced by factor 5 compared with pure TLM simulation. Applying HOS methods we are expecting a further improvement of accuracy and also a reduction of the computational effort. A decreasing memory and computation time of TLM analysis allows us to simulate more complex structures.

V. References

[1] P.B. Johns, R.L. Beurle, "Numerical Solution of 2-Dimensional Scattering Problems using a Transmission-Line Matrix" *Proc. IEE*, vol.118, no. 9, pp 1203-1208, Sept. 1971.

[2] P. Russer, M. Righi, C. Eswarappa and W.J.R. Hoefer. "Lumped Element Equivalent Circuit Parameter Extraction of Distributed Microwave Circuits via TLM Simulation," 1994 MTT-S International Microwave Symposium Digest, pp. 887–890, 1994.

[3] Ch. Eswarappa, W. J. R. Hoefer, "Autoregressive (AR) and Autoregressive Moving Average (ARMA) Spectral Estimation Techniques for faster TLM Analysis of Microwave Structures", IEEE Trans. on Microwave Theory and Techniques, Vol. 42, No. 12, pp. 2407 ff., Dec. 1994. 3. C.K.

[4] C. Nikias, A. Petropulu, "Higher-Order Spectra Analysis", New Jersey, 1993.

[5] V. Belevitch, "Classical Network Theory," Holden–Day, San Francisco, 1968.

[6] T. Mangold and P. Russer, "Full-Wave Modelling and Automatic Equivalent-Circuit Generation of Millimeter-Wave Planar and Multilayer Structures", IEEE Trans. on Microwave Theory and Techniques, 47, (6), pp. 851-858, June 1999.

[7] L. Vietzorreck, F. Coccetti, V. Chtchekatourov, P. Russer, "Modeling of MEMS Capacitive switches by TLM", 2000 IEEE MTT-S International Microwave Symposium Digest, Volume 2, Boston, MA, 11-16 June 2000, pp.1221-1223.

[8] L. Ljung, "System Identification. Theory for the User", Prentice Hall PTR,Upper Saddle River, NJ, 1999.

[9] S. Lawrence Marple Jr.,"Digital Spectral Analysis", Prentice Hall, Inc., Englewood Cliffs, NJ, 1987.

[10] E. K. Miller and T. K. Sarkar, "An Introduction to the use of Model-Based Parameter Estimation in Electromagnetics", in Review of Radio Science, 196-99, ed. by R. Stone, Oxford University Press, August 1999, pp. 139-174.

[11] V. K. Jain and T. Sarkar, "Multivariable System Identification by Pencil-of-Function Method", IEEE Trans. on Instrumentation and Measurement, Vol. IM-34, No.4., Dec.1985.

[12] V. Milanovic, M. Maharbiz, and K.S.J Pister, "Batch Transfer Integration of RF Microrelays", 2000 IEEE Microwave and Guided Wave Letters, Volume 10, No.8, August 2000, pp. 313-315.